

# Feasibility study on prediction of properties of municipal solid waste with time series models

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## Abstract

Chemical properties of solid waste (SW) are the important design parameters for municipal incineration plant projects. In Taiwan, a frequency analysis has been used to predict values and trend of variation of SW properties. Yet few acceptable results were found, especially in municipal areas, mainly due to the changing properties of SW affected by living standards, consumer habits, degree of implementation of resource recovery and other complicated factors. In this study, the historical monthly mean values (July 1988–April 1995) of some chemical properties of SW of the Taipei City were analyzed with time series analysis methodology which usually provides more accurate information about the trend of historic data. Suitable time series models have been successfully developed to predict chemical properties including gross heat value (GHV), low heat value (LHV), water content, combustible content, ash content and element composition of C, H, N, O, S, Cl in categories of general SW, downtown SW, suburb SW and market SW. The mean absolute percentage errors (MAPE) of these models are usually calculated as less than 20% indicating that these models have good predicting ability. © 1998 Elsevier Science B.V.

*Keywords:* Time series models; Solid waste; Incineration

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## 1. Introduction

Incineration of solid waste (SW) has become a key solution for SW treatment and disposal over the past decade in Taiwan, especially in municipal areas due to the dramatically increasing difficulty in finding suitable sites for traditional sanitary landfill.

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Thus, the reliability of prediction of SW chemical property values, including heat values (GHV and LHV), the three components (water content, combustible content and ash content) and the six major elements (C, H, N, O, S, Cl), has become more and more important since they are not only related to the designed capacity of the incineration plant, but also to the heat recovery efficiency of boilers. In most cases, such design values were determined based on simple approaches such as linear regression analysis, curve extending by eyes or a normal property distribution analysis. However, the SW properties determined by these means seldom fitted the data, especially the LHV values resulting in low operating performance of the incineration plants. For example, the LHV of the Nei-hu incineration plant of the Taipei City was designed as 1300 kcal/kg, while, the operating LHV was later found to be over 1700 kcal/kg during the operation phase. Accordingly, SW of 700 ton/d or less than 76% of the design capacity of this plant was allowed to be performed. Difficulties in predicting reliable SW chemical properties arise mainly from the lack of representative SW samples for lab analysis as well as the irregular variation of the SW properties which are easily affected by the living standards, consumer habits, commercial activities, degree of refuse resource recovery, etc.

To overcome the difficulties in predicting the SW chemical properties, the methodology of Time Series Analysis was applied in this study because this analysis approach can usually predict the trend of change of a time series with a better reliability mainly because it provides useful information about the inter-relationship between adjacent data in a time series. Time series prediction models have been successfully developed in many other fields, but this is the first study to apply this modeling technique to predict SW properties in Taiwan.

## 2. Materials and methods

### 2.1. Database

In this study, each time series comprises 82 consecutive monthly means of a SW chemical property dating from July 1988 to April 1995. Selected SW chemical properties include GHV, LHV, the three components and the six major elements. Solid waste of the Taipei City was also divided into four categories as general SW, downtown SW, suburb SW and market SW [1–7].

### 2.2. Time series predicting models

The well-known Autoregressive Moving Average Model (ARMA) proposed by G.E.P. Box and G.M. Jenkins in 1970 was applied through this study [8]. The ARMA model takes the form:

$$\psi_p(B)(1-B)^d Y_t = C + \theta_q(B) e_t \quad (1)$$

where,  $C$  = constant,  $Y_t$  = observation in time  $t$ ,  $e_t$  = residual in time  $t$ ,  $p, d, q$  = orders of the model,  $B$  = backshift operator,  $BY_t = Y_{t-1}$ ,  $\psi_p(B) = 1 - \psi_1 B - \psi_2 B^2 - \dots - \psi_p B^p$ ,  $\theta_q(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$ ,  $\psi_1, \dots, \psi_p$  are autoregression parameters and  $\theta_1, \dots, \theta_q$  are the moving average parameters.

### 2.3. Model fitting procedure

#### 2.3.1. Data pre-analysis

Arrange the collected data as figures in the form of time series (one plot of the gross heat values vs. sampling time is presented in Fig. 1 as an example), then justify whether the series is a stationary series or a non-stationary series by eyes. If the series shows non-stationarity, the data needs to be transformed to stationarity by differencing.

#### 2.3.2. Model identification

2.3.2.1. *Determination of  $d$ .* If a non-stationary series is observed, the data need to be differenced till the Autocorrelation Function (ACF) of the series reduced significantly. The  $d$  value stands for the order of the difference which reduces the series to stationarity.

2.3.2.2. *Determine the  $p$  and  $q$ .* Once the  $d$  value is determined, the figures of the ACF and the PACF (Partial autocorrelation Function) of the series are generated and analyzed to determine the best-fit  $p$  and  $q$  values as follows.

(1) MA models (moving average models). If the variable at time  $t$  can be expressed as the sum of residuals at previous times, then it may be represented by an MA model of the form.

$$Y_t = C + \theta_q(B) e_t \tag{2}$$

To justify an MA( $q$ ) model, the series of ACF values should reduced gradually and substantially disappear after a time lag of  $q$ .

(2) AR models (autoregression models). If the variable at time  $t$  can be expressed as the sum of the weighted variables at previous time, then it may be represented by an AR model of the form:

$$\psi_p(B) Y_t = C + e_t \tag{3}$$

Justification of an AR( $p$ ) series needs a series of gradually reducing ACF values and the PACF values should disappear after time lag  $p$ .

(3) ARMA series (autoregression moving average series). If a variable of time  $t$  can be expressed as the sum of weighted observations and weighted residuals of previous time, then it may be represented by an ARMA model of the form:

$$\psi_p(B) Y_t = C + \theta_q(B) e_t \tag{4}$$

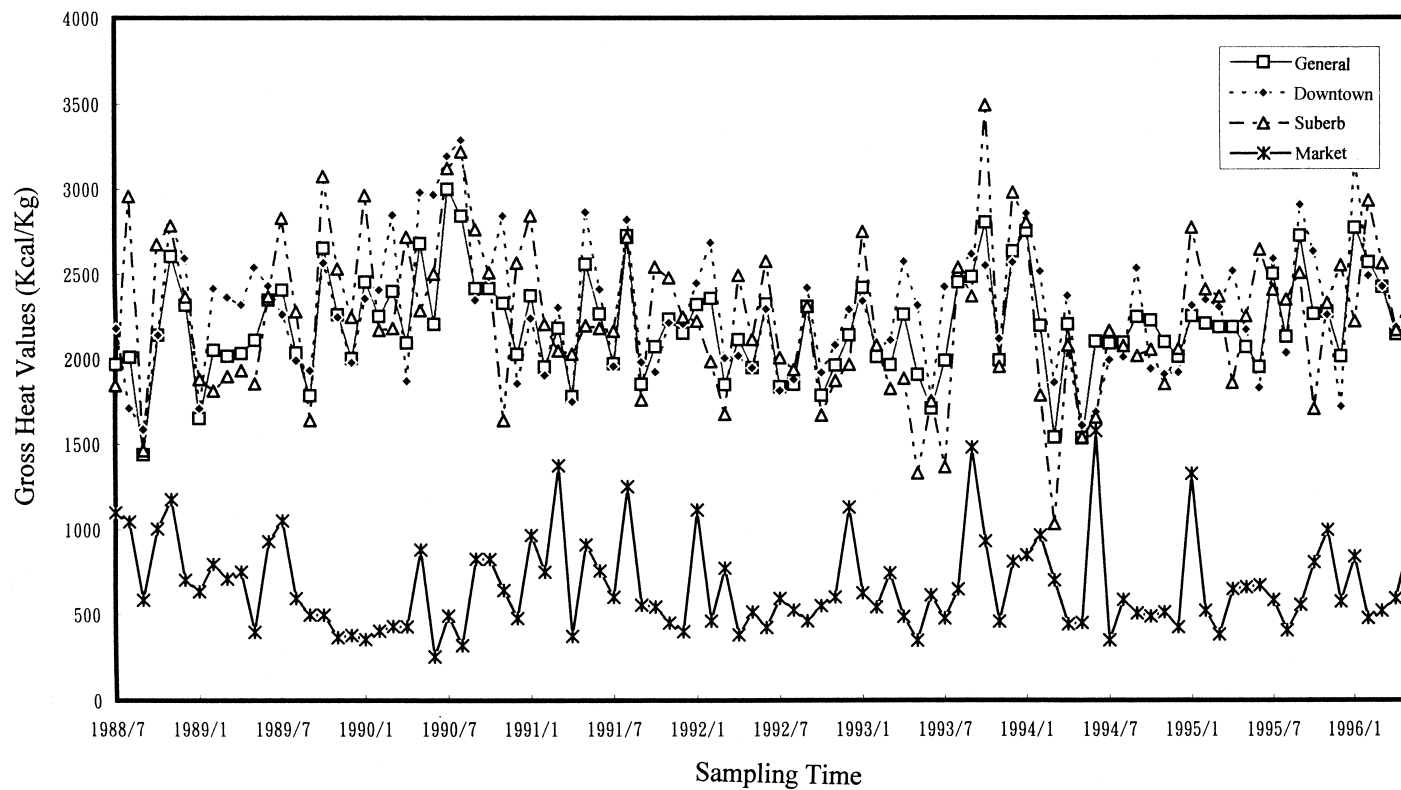


Fig. 1. Variation of monthly mean gross heat values of Taipei municipal solid waste.

Table 1  
Summary of prediction models for general SW of the Taipei City

GHV (kcal/kg)	Model: $(1 - \psi_1 B)Y_t = (1 - \theta_1 B)e_t$				
	Parameter	Estimate	Std. error	t-value	lag
	$\psi_1$	0.8594	0.0604	1789.22	1
	$\theta_1$	0.9998	0.0006	14.23	1
LHV (kcal/kg)	Model: $(1 - \psi_1 B)Y_t = (1 - \theta_1 B)e_t$				
	$\psi_1$	0.9997	0.0009	1109.13	1
	$\theta_1$	0.8658	0.0595	14.56	1
S (%)	Model: $(1 - \psi_1 B)Y_t = e_t$				
	$\psi_1$	-0.4983	0.0964	-5.17	1
Cl(%)	Model: $(1 - B^2)(1 - \psi_2 B^2 - \psi_4 B^4)Y_t = (1 - \theta_4^4)e_t$				
	$\psi_2$	-0.4352	0.1398	-3.11	2
	$\psi_4$	-0.4149	0.1814	2.29	4
	$\theta_4$	0.9997	67.2689	2.01	4
Water (%)	Model: $(1 - \psi_1 B)Y_t = (1 - \theta_1 B)e_t$				
	$\psi_1$	0.9999	0.0002	16.35	1
	$\theta_1$	0.8957	0.0548	5954.12	1
Ash (%)	Model: $(1 - \psi_1 B)Y_t = (1 - \theta_1 B)e_t$				
	$\psi_1$	0.9998	0.0004	2431.91	1
	$\theta_1$	0.9315	0.0448	20.79	1
Combustible (%)	Model: $(1 - \psi_1 B)Y_t = (1 - \theta_1 B)e_t$				
	$\psi_1$	0.9997	0.0003	3489.64	1
	$\theta_1$	0.8446	0.0620	13.62	1
C (%)	Model: $(1 - \psi_1 B)Y_t = (1 - \theta_1 B)e_t$				
	$\psi_1$	1.000	0.0001	9999.99	1
	$\theta_1$	0.9905	0.1458	6.79	1
H (%)	Model: $(1 - \psi_1 B)Y_t = (1 - \theta_1 B)e_t$				
	$\psi_1$	1.0000	1.6575E-9	9999.99	1
	$\theta_1$	0.5711	0.1121	5.09	1
N (%)	Model: $(1 - B)(1 - \psi_3 B^3)Y_t = (1 - \theta_1 B)e_t$				
	$\theta_3$	-0.3016	0.1111	-2.71	3
	$\theta_1$	0.6464	0.0901	7.18	1
O (%)	Model: $(1 - \psi_2 B^2 - \psi_3 B^3)Y_t = (1 - \theta_3 B^3)e_t$				
	$\theta_3$	0.7689	0.0801	9.61	3
	$\theta_3$	0.5927	0.1161	5.10	3

Justification of an ARMA model needs the reducing trend of the ACF and PACF values however the  $p$  and  $q$  values are not quite so readily determined.

2.3.2.3. *Estimation of parameters.* After the type of model has been determined, the parameters of the model can be estimated using maximum likelihood methods.

2.3.3. *Model testing*

The model testing processes include a hypothesis test procedure (the critical value of  $t$  was 1.96 at a 95% C.I.) to test the significance of the estimated parameters of each model and a white noise checking process of the normal distribution of the residuals to check if the residual distribution is normal with a mean close to zero. If the residuals form white noise process, the model can be accepted.

2.3.4. *Model prediction*

Model prediction ability was assessed in terms of the mean absolute percentage error (MAPE) defined as:

$$MAPE = \frac{1}{n} \sum_{t=1}^n \frac{|e_t|}{Y_t} \times 100 \tag{5}$$

where,  $e_t = Y_t - \hat{Y}_t$ ,  $Y_t$  is the observation,  $\hat{Y}_t$  is the estimate, and  $n$  is the number of observations. The prediction ability of the models would be classified as excellent, good,

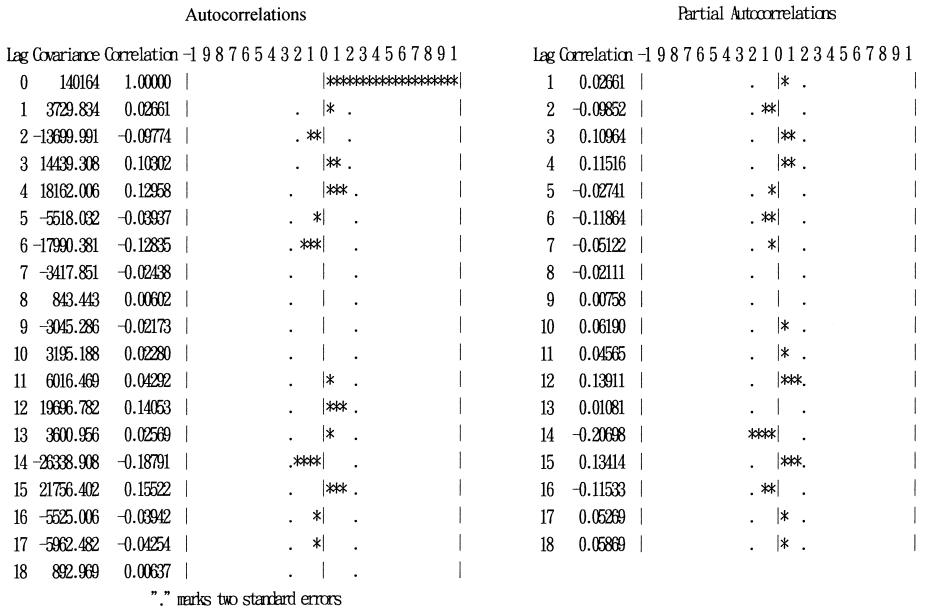


Fig. 2. ACF and PACF analysis of GHV residuals for the general SW.

Autocorrelation Check for White Noise

To	Chi	Autocorrelations							
Lag	Square	DF	Prob						
6	4.92	6	0.554	0.027	-0.098	0.103	0.130	-0.039	-0.128
12	7.20	12	0.844	-0.024	0.006	-0.022	0.023	0.043	0.141
18	13.67	18	0.750	0.026	-0.188	0.155	-0.039	-0.043	0.006

Fig. 3. Autocorrelation check for white noise of GHV residuals for the general SW.

acceptable or poor if the MAPE values are < 10, 10–20, 20–50, or > 50, respectively. In this study, the monthly mean values of each of the SW chemical properties during the period of May 1994 to April 1995 were selected as the observations and were compared with the values predicted by the fitted time series models using data dating from July 1988 to April 1994.

3. Results

Due to limitation of space, only parts of the original results of this study are presented here including the fitted time series for the general SW of the Taipei City (Table 1), some typical ACF and PACF figures and checks for residual white noise analysis for the GHV, LHV, and water content of the general SW (Figs. 2–7).

Autocorrelations													Partial Autocorrelations																					
Lag	Covariance	Correlation	-1	9	8	7	6	5	4	3	2	1	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9			
0	132571	1.00000												1	0.05020																			
1	6654.431	0.05020												2	-0.05516																			
2	-6659.588	-0.05250												3	0.11459																			
3	14370.723	0.10840												4	0.07816																			
4	12209.348	0.09210												5	-0.08961																			
5	-12029.229	-0.09074												6	-0.14199																			
6	-19135.355	-0.14434												7	-0.06423																			
7	-6329.998	-0.04775												8	0.01249																			
8	966.665	0.00729												9	0.02783																			
9	-2139.911	-0.01614												10	0.05567																			
10	2910.042	0.02195												11	0.03012																			
11	6343.530	0.04785												12	0.09545																			
12	16250.338	0.12255												13	0.02124																			
13	6487.473	0.04894												14	-0.22049																			
14	-25835.365	-0.19488												15	0.13437																			
15	17223.600	0.12992												16	-0.08553																			
16	-5055.507	-0.03813												17	0.07332																			
17	-3882.592	-0.02929												18	0.02513																			
18	-2281.551	-0.01721																																

"," marks two standard errors

Fig. 4. ACF and PACF analysis of the LHV residuals for the general SW.





## 4. Discussion

### 4.1. GHV models

The MAPE values of the GHV models for the general SW, down SW and suburb SW are 5.4, 13.5 and 12.0, respectively (Table 2) indicating the good prediction ability of these models. However, the MAPE value of the market SW is only 42.5 implying it is only an acceptable model. The lack of goodness of fit of this model may be due to the large variation in the chemical properties of the market SW. The prediction shows a slightly reducing trend of the GHV in the near future implying the implementation of recovery of high heat value waste (e.g. papers, plastics) have already reduced the gross heat value of SW of Taipei City substantially and will continue to it.

Table 2  
Prediction of GHV for the general SW of the Taipei City (kcal/kg)

Time	Real obs.	Predicted	Std. error	Lower limit of 95%	Upper limit of 95%	MAPE
1994/5	1533	2198	310	1590	2806	5.4
1994/6	2101	2104	310	1497	2712	
1994/7	2090	2103	310	1496	2711	
1994/8	2095	2101	310	1493	2709	
1994/9	2245	2100	310	1492	2708	
1994/10	2225	2120	310	1512	2728	
1994/11	2098	2134	310	1526	2742	
1995/1	2251	2112	310	1504	2719	
1995/2	2206	2131	310	1523	2739	
1995/3	2185	2141	310	1533	2749	
1995/4	2185	2147	310	1539	2754	
1995/5		2152	310	1544	2759	
1995/6		2152	313	1537	2765	
1995/7		2151	316	1531	2770	
1995/8		2150	319	1525	2776	
1995/9		2150	322	1518	2781	
1995/10		2149	325	1512	2786	
1995/11		2149	328	1506	2792	
1995/12		2148	331	1500	2797	
1996/1		2148	334	1494	2802	
1996/2		2148	337	1488	2807	
1996/3		2147	339	1482	2812	
1996/4		2147	342	1476	2817	
1996/5		2146	345	1470	2822	
1996/6		2146	348	1465	2827	
1996/7		2145	350	1459	2832	
1996/8		2145	353	1453	2837	
1996/9		2145	356	1448	2842	
1996/10		2144	358	1442	2846	
1996/11		2144	361	1436	2851	
1996/12		2143	363	1431	2856	

Table 3  
 Prediction of LHV for the general SW of the Taipei City (kcal/kg)

Time	Real obs.	Predicted	Std. error	Lower limit of 95%	Upper limit of 95%	MAPE
1994/5	1044	1749	328	1106	2392	
1994/6	1590	1654	328	1011	2297	
1994/7	1619	1645	328	1002	2288	
1994/8	1641	1641	328	998	2284	
1994/9	1821	1640	328	998	2283	
1994/10	1767	1664	328	1021	2307	7.2
1994/11	1658	1677	328	1035	2320	
1994/12	1564	1674	328	1031	2317	
1995/1	1795	1659	328	1016	2301	
1995/2	1753	1676	328	1034	2319	
1995/3	1732	1686	328	1043	2329	
1995/4	1700	1691	328	1049	2334	
1995/5		1692	328	1050	2335	
1995/6		1691	331	1043	2340	
1995/7		1691	338	1037	2345	
1995/8		1690	337	1031	2350	
1995/9		1690	339	1025	2355	
1995/10		1689	342	1019	2360	
1995/11		1689	345	1013	2365	
1995/12		1688	348	1007	2370	
1996/1		1688	351	1001	2375	
1996/2		1687	353	995	2380	
1996/3		1687	356	989	2384	
1996/4		1686	359	983	2389	
1996/5		1685	361	977	2394	
1996/6		1685	364	971	2398	
1996/7		1684	367	966	2403	
1996/8		1684	369	960	2407	
1996/9		1683	372	955	2412	
1996/10		1683	374	949	2416	
1996/11		1682	377	943	2421	
1996/12		1681	379	938	2425	

#### 4.2. LHV models

Good prediction abilities were also found for the LHV models for the general SW, downtown SW and suburb SW, with MAPE values of 7.2, 18.3 and 6.6, respectively (Table 3). These models also predict slightly reducing trends of LHV for all types of SW. Again, the MAPE value of 49.7 of the LHV model for the market SW might be due to the large variation, especially the water content.

#### 4.3. The three component models

The MAPE vales of the water, combustibile and ash component models for the general SW are calculated as 6.4, 5.3 and 28.2 (Table 4), respectively showing good

prediction ability of these models. Similar good results were obtained for the downtown SW and suburb SW but not for the market SW. However all the MAPE values are less than 50 (acceptable). The relatively poor prediction ability of the ash content model may be due to the low sensitivity of ash analysis since the measured ash amount is much less than the other two components.

#### 4.4. The six element models

For the general SW, the MAPE values of the models of the six elements vary significantly from 5.2 (C), 6.6 (H), 8.6 (O), 23.3 (S), 29.6 (N) to 55.5 (Cl). Similar results were observed for other types of SW. The difference in the MAPE values between these elements may due to the degree of variation of the data series. The time series for the Cl was found to be the most non-stationary.

## 5. Conclusions

In this study, suitable time series models for predicting the GHV, LHV, the three components and the six major elements (C, H, N, O, S, Cl) have been developed. The prediction ability of these models was generally found to be excellent or good, except for the ash content, element Cl content and for other chemical properties of the market SW. The difference in the MAPE values of the fitted models was thought to be related to the degree of variation of the time series under consideration.

This study demonstrates that the ARMA time series models can be successfully used for the prediction of chemical properties of municipal solid waste. However, to obtain better prediction results, the parameters of these models should be re-evaluated once new data are obtained.

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